Lecture 7: ELEMENTS

TECHSummer School at WUST, Steel Structures 22-23.07.2019

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ELEMENTS

Lecture 7.9.1 : Unrestrained Beams I

SUMMARY: This lecture is restricted to beams whose design may be based on simple strength of materials considerations. Behaviour in simple bending is discussed, leading to the concept of section modulus as the basis for strength design. Subsidiary considerations of shear strength, resistance to local loads and adequate stiffness against deflection are also mentioned. Behaviour under complex loading, producing bending about both principal axes, or combined bending and torsion is introduced.

ADDITIONAL NOTATION

C coefficient to account for type of loading

d overall depth

EI_z flexural rigidity about the minor axis

f_d design strength of material

f_y material yield strength

i_z minor axis radius of gyration

k coefficient to account for conditions of lateral support

L span

$M_{b.Rd}$ buckling resistance moment

M_{cr} elastic critical buckling moment

M_{pl} plastic moment of cross-section

M_{Rd} moment resistance of cross-section

t_f flange thickness

u lateral deflection

α_{LT} parameter in design formula, see Equation (2)

χ_{LT} reduction factor for lateral-torsional buckling

λ_{LT} beam slenderness

λ_{LT} basic slenderness

parameter used to determine $_{LT}$, see Equation (4)

ϕ twist

ϕ_{LT} parameter used to determine χ_{LT}, see Equation (2)

ψ moment ratio, see Equation (5)

1. INTRODUCTION

When designing a steel beam it is usual to think first of the need to provide adequate strength and stiffness against vertical bending. This leads naturally to a cross-sectional shape in which the stiffness in the vertical plane is much greater than that in the horizontal plane. Sections normally used as beams have the majority of their material concentrated in the flanges, which are relatively narrow so as to prevent local buckling. The need to connect beams to adjacent members with ease normally suggests the use of an open section, for which the torsional stiffness will be comparatively low. Figure 1, which compares section properties for four different shapes of equal area, shows that the high vertical bending stiffness of typical beam sections is obtained at the expense of both horizontal bending and torsional stiffness.

1. INTRODUCTION

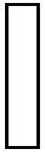
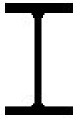
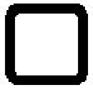
Section type	Flat	H-Sections (Typical)	I-Sections (Typical)	Hollow sections (Typical)
Section properties				
A Area	1	1	1	1
I_y (Vertical loading)	1	0,35	1	0,2
I_z (Horizontal loading)	0,2	3,5	1	3,5
J (Twisting)	1	1	1	100

Figure 1 Types of cross-section used as beams showing relative values of section properties

2. RESPONSE OF SLENDER BEAMS TO VERTICAL LOADING

It is known from our understanding of the behaviour of struts that, whenever a slender structural element is loaded in its stiff plane (axially in the case of the strut), there exists a tendency for it to fail by buckling in a more flexible plane (by deflecting sideways in the case of the strut). Figure 2 illustrates the response of a slender cantilever beam to a vertical end load; this phenomenon is termed lateral-torsional buckling. Although it involves both a lateral deflection (u) and twisting about a vertical axis through the web (f), as shown in Figure 3, this type of instability is quite similar to the simpler flexural buckling of an axially loaded strut. Loading the beam in its stiffer plane (the plane of the web) has induced a failure by buckling in a less stiff-direction (by deflecting sideways and twisting).

2. RESPONSE OF SLENDER BEAMS TO VERTICAL LOADING

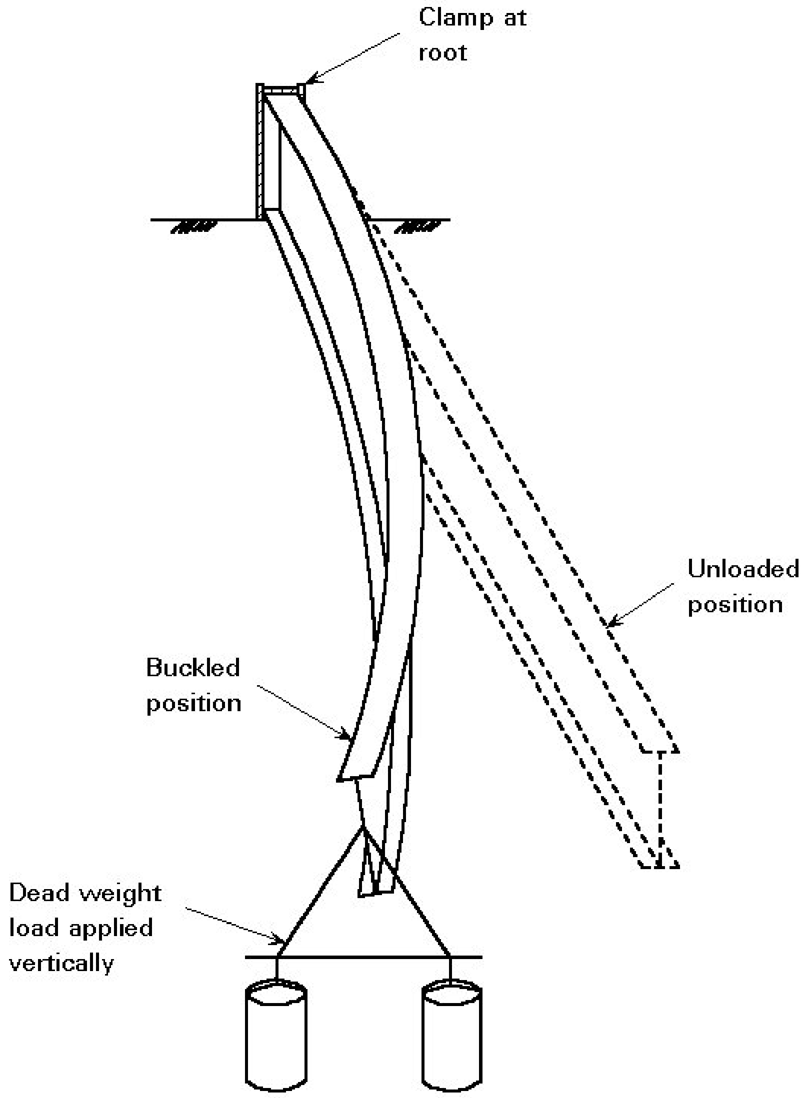


Figure 2 Response of a slender cantilever beam to vertical loading : lateral-torsional buckling

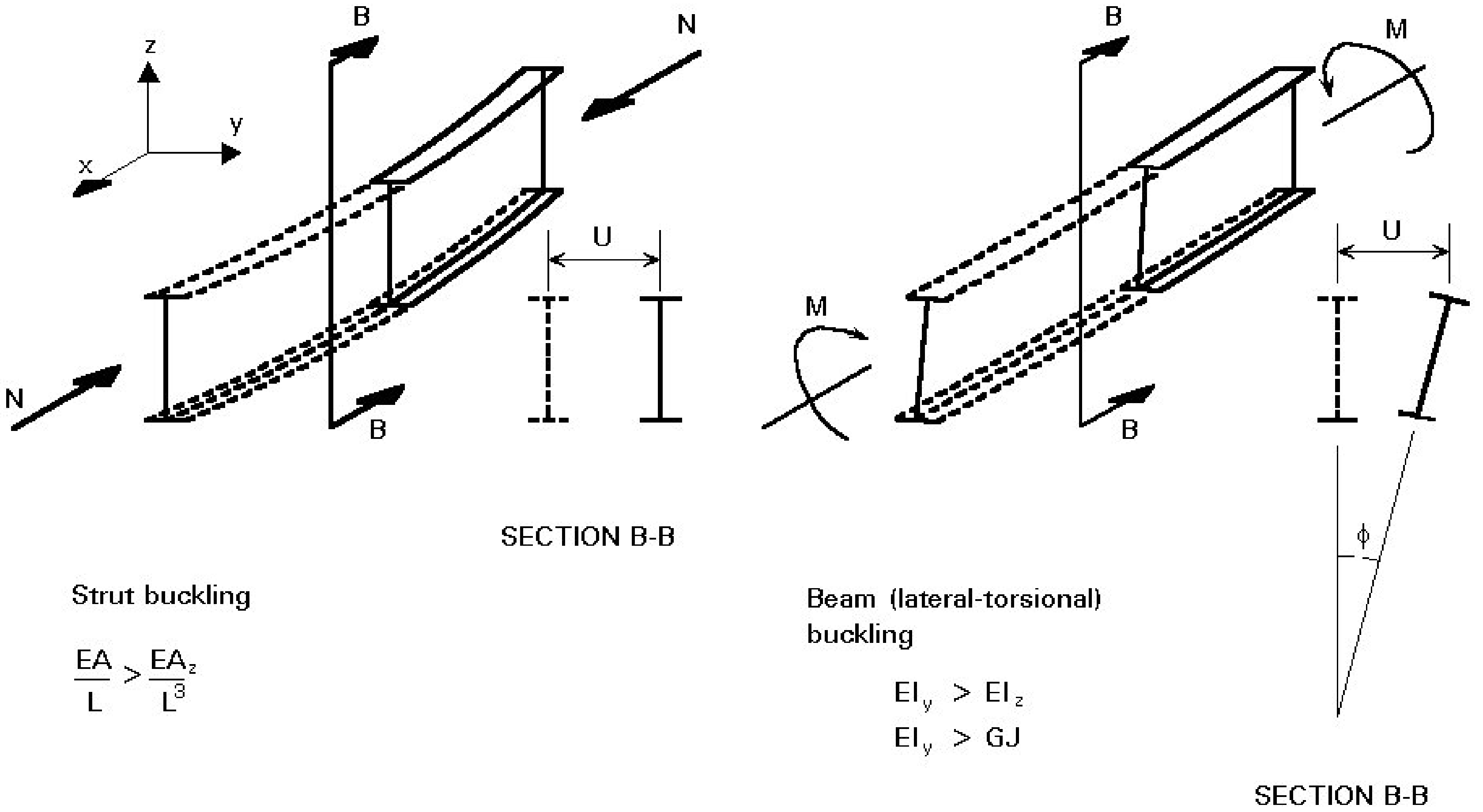


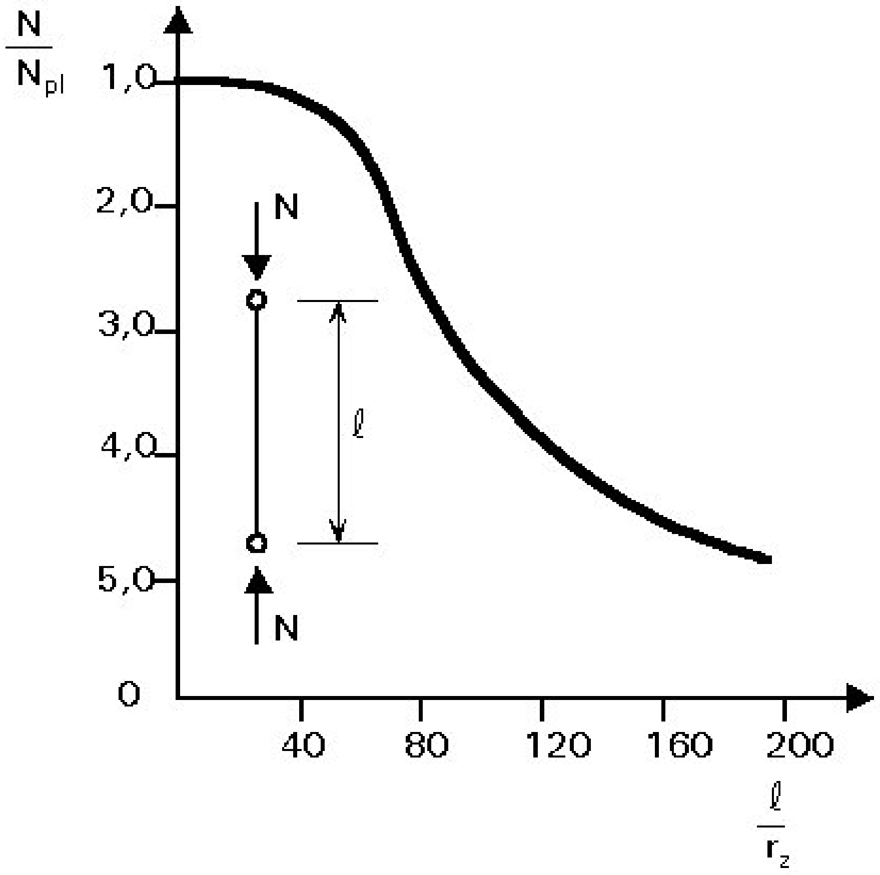
Figure 3 Similarity between strut buckling and beam buckling

2. RESPONSE OF SLENDER BEAMS TO VERTICAL LOADING

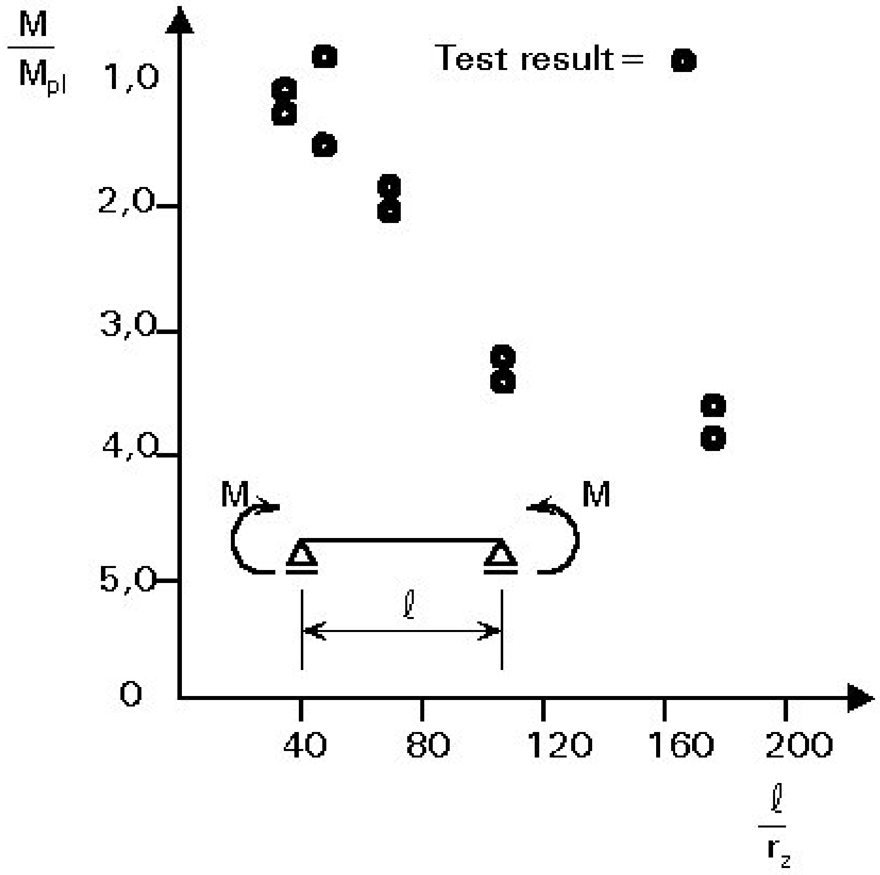
Of course, many types of construction effectively prevent this form of buckling, thereby enabling the beam to be designed with greater efficiency as fully restrained (see Lecture 7.8.1). In this context it is important to realise that during erection of the structure certain beams may well receive far less lateral support than will be the case when floors, decks, bracings, etc., are present, so that stability checks, at this stage, are also necessary.

Lateral-torsional instability influences the design of laterally unrestrained beams in much the same way that flexural buckling influences the design of columns. Thus the bending strength will now be a function of the beam's slenderness, as indicated in Figure 4, requiring the use in design of an iterative procedure similar to the use of column curves in strut design. However, because of the type of structural actions involved, the analysis of lateral-torsional buckling is considerably more complex. This is reflected in a design approach which requires a rather greater degree of calculation.

2. RESPONSE OF SLENDER BEAMS TO VERTICAL LOADING



Typical strut design curve in ultimate load format



Test data for 305 x 102 UB 28 sections as laterally unrestrained beams

Figure 4 Dependence of beam strength on unrestrained length and analogy with column strength

3. SIMPLE PHYSICAL MODEL

Before considering the analysis of the problem, it is useful to attempt to gain an insight into the physical behaviour by considering a simplified model. Since bending of an I-section beam is resisted principally by the tensile and compressive forces developed in two flanges, as shown in Figure 5, the compression flange may be regarded as a strut. Compression members exhibit a tendency to buckle and in this case the weaker direction would be for the flange to buckle downwards. However, this is prevented by the presence of the web. Therefore the flange is forced to buckle sideways, which will induce some degree of twisting in the section as the web too is required to deform. Whilst this approach neglects the real influence of torsion and the role of the tension flange, it does, nevertheless, approximate the behaviour of very deep girders with very thin webs or of trusses or open web joists. Indeed, early attempts at analysing lateral-torsional buckling started with this approach.

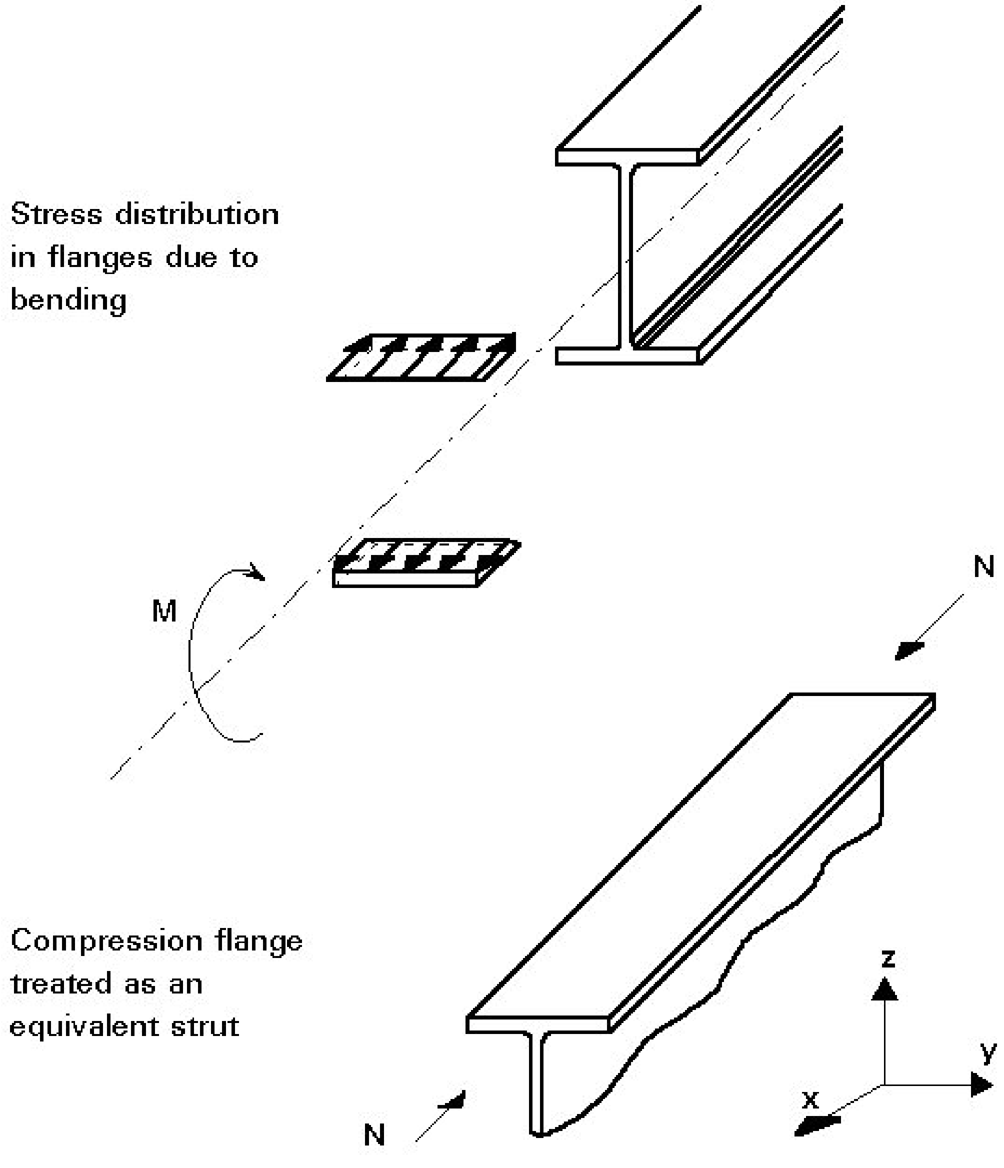


Figure 5 Approximation of beam buckling problem as a strut problem

4. FACTORS INFLUENCING LATERAL STABILITY

The compression flange/strut analogy, discussed in the previous section, is also helpful in understanding the following:

1. The buckling load of the beam is likely to be dependent on its unbraced span, i.e.the distance between points at which lateral deflection is prevented, and on its lateral bending stiffness (EL_z) because strut resistance $\mu EL_z/L^2$.
2. The shape of the cross-section may be expected to have some influence, with the web and the tension flange being more important for relatively shallow sections, than for deep slender sections. In the former case the proximity of the stable tension flange to the unstable compression flange increases stability and also produces a greater twisting of the cross-section. Thus torsional behaviour becomes more important.

4. FACTORS INFLUENCING LATERAL STABILITY

3. For beams under non-uniform moment, the force in the compression flange will no longer be constant, as shown in Figure 6. Therefore such members might reasonably be expected to be more stable than similar members under a more uniform pattern of moment.

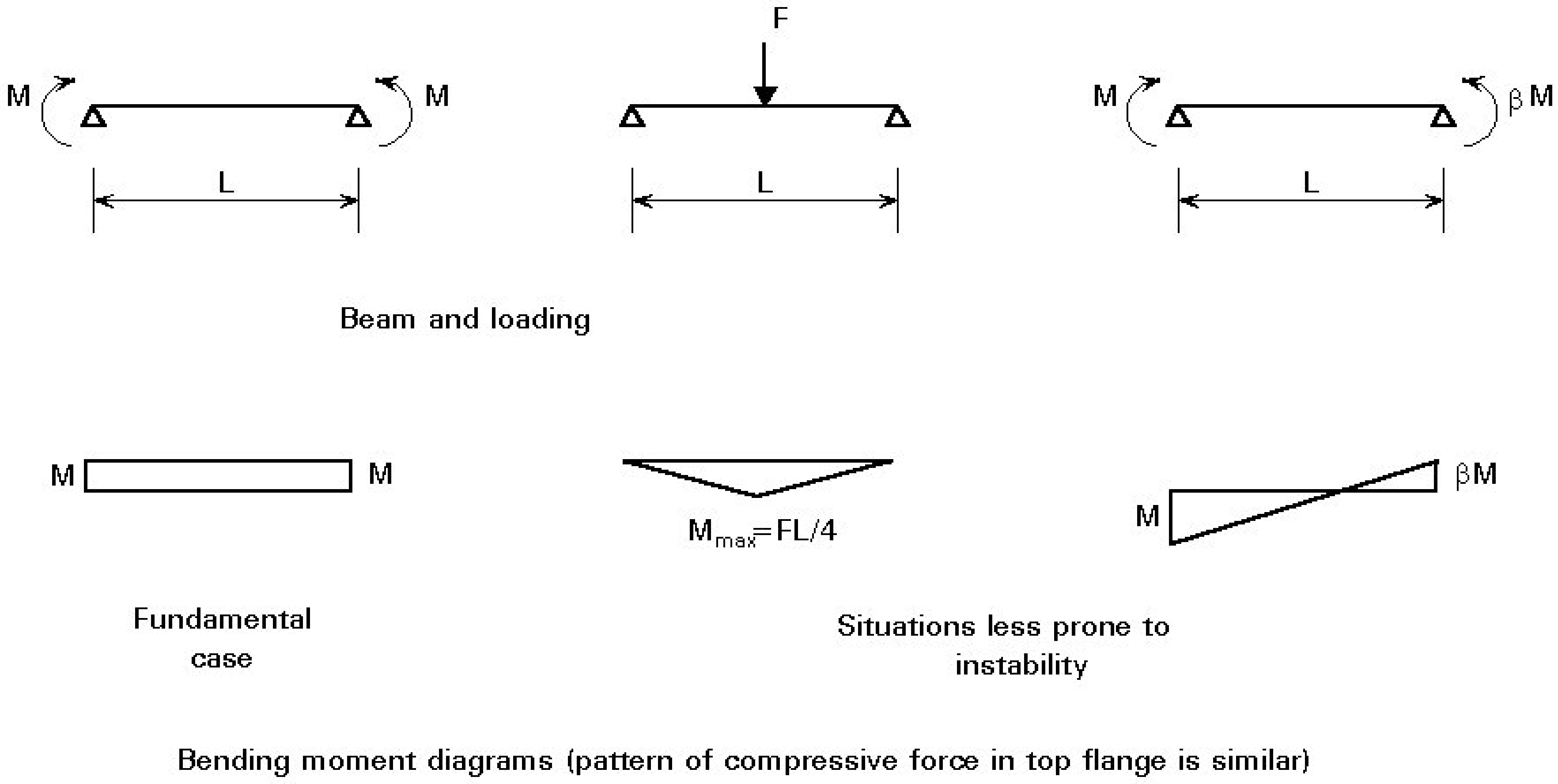


Figure 6 Effect of non-uniform moment on lateral-torsional buckling

4. FACTORS INFLUENCING LATERAL STABILITY

4. End restraint which inhibits development of the buckled shape, shown in Figure 3, is likely to increase the stability of the beam. Consideration of the buckling deformations (u and f) should make it clear that this refers to rotational restraint in plan, i.e.about the z-axis (refer back to Figure 5 and 3). Rotational restraint in the vertical plane affects the pattern of moments in the beam (and may thus also lead to increased stability) but does not directly alter the buckled shape, as shown in Figure 7.

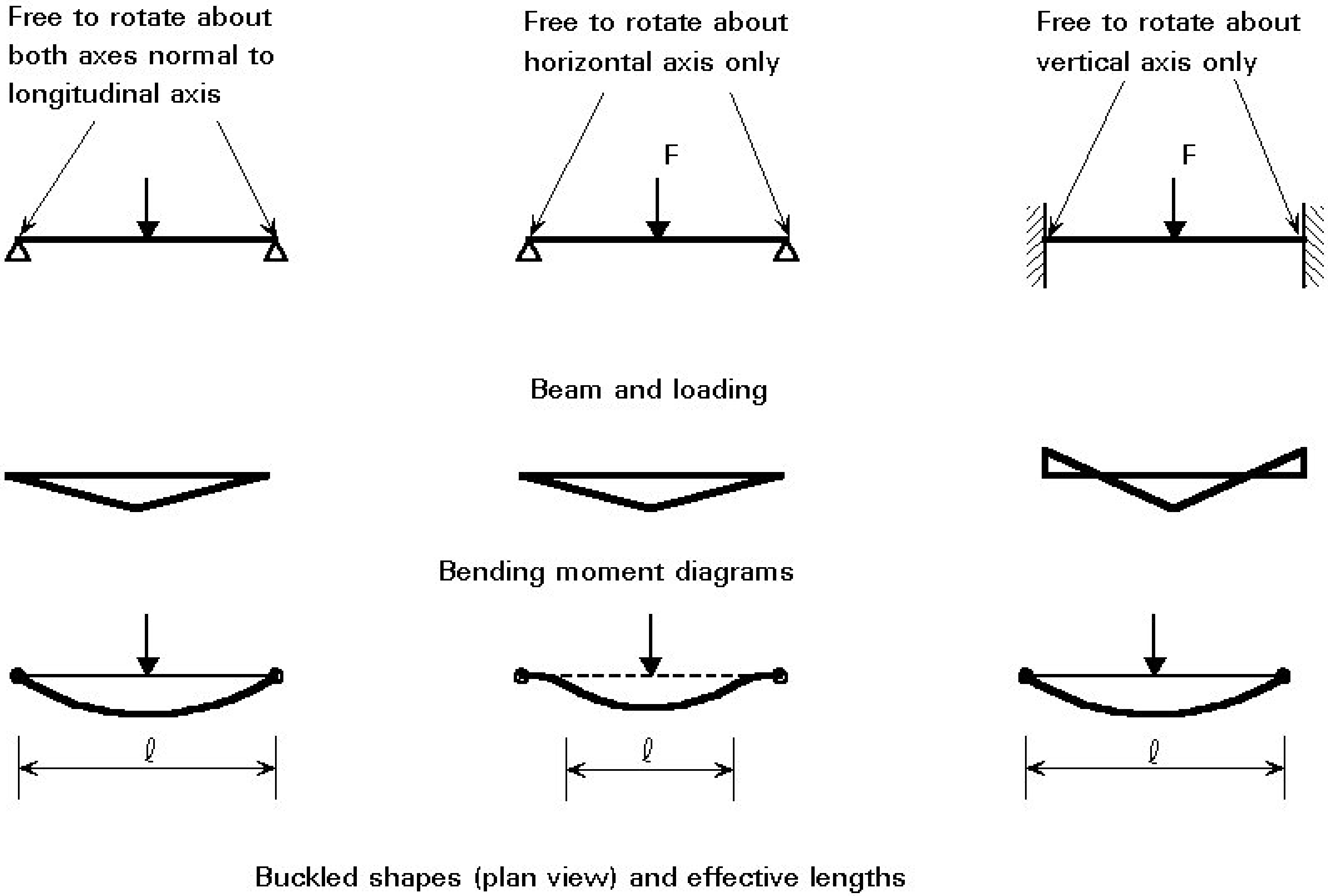


Figure 7 Effect of end restraint in plan or elevation on lateral-torsional buckling

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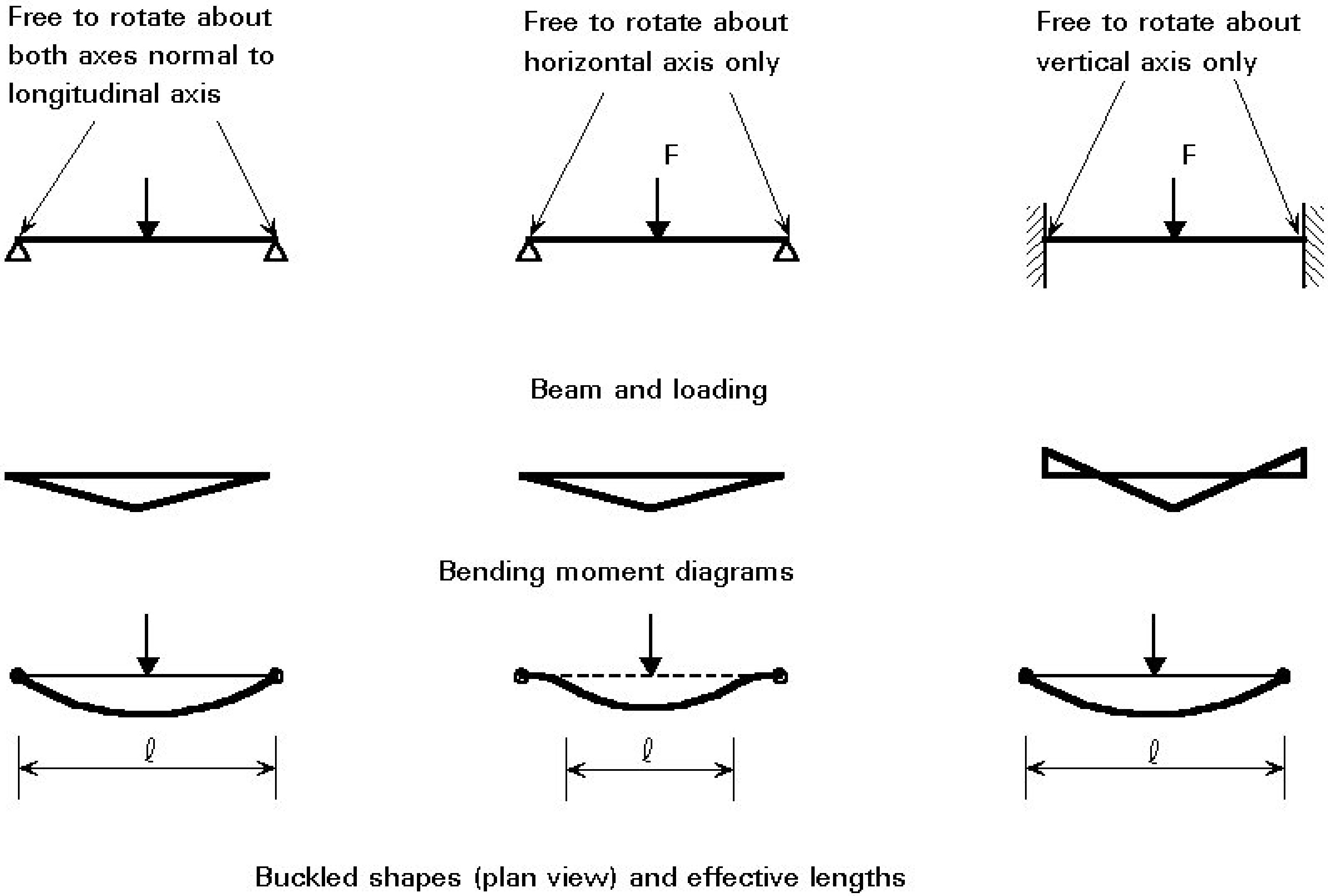


Figure 7 Effect of end restraint in plan or elevation on lateral-torsional buckling

5. BRACING AS A MEANS OF IMPROVING PERFORMANCE

Bracing may be used to improve the strength of a beam that is liable to lateral-torsional instability. Two requirements are necessary:

The bracing must be sufficiently stiff to hold the braced point effectively against lateral movement (this can normally be achieved without difficulty).

The bracing must be sufficiently strong to withstand the forces transmitted to it by the main member (these forces are normally a percentage of the force in the compression flange of the braced member).

5. BRACING AS A MEANS OF IMPROVING PERFORMANCE

Providing these two conditions are satisfied, then the full in-plane strength of a beam may be developed through braces at sufficiently close spacing. Figure 8, which illustrates buckled shapes for beams with intermediate braces, shows how this buckling involves the whole beam. In theory, bracing should prevent either lateral or torsional displacement from occurring. In practice, consideration of the buckled shape of the beam cross-section shown in Figure 3 suggests that bracing is potentially most effective when used to resist the largest components of deformation, i.e. a lateral brace attached to the top flange is likely to be more effective than a similar brace attached to the bottom flange.

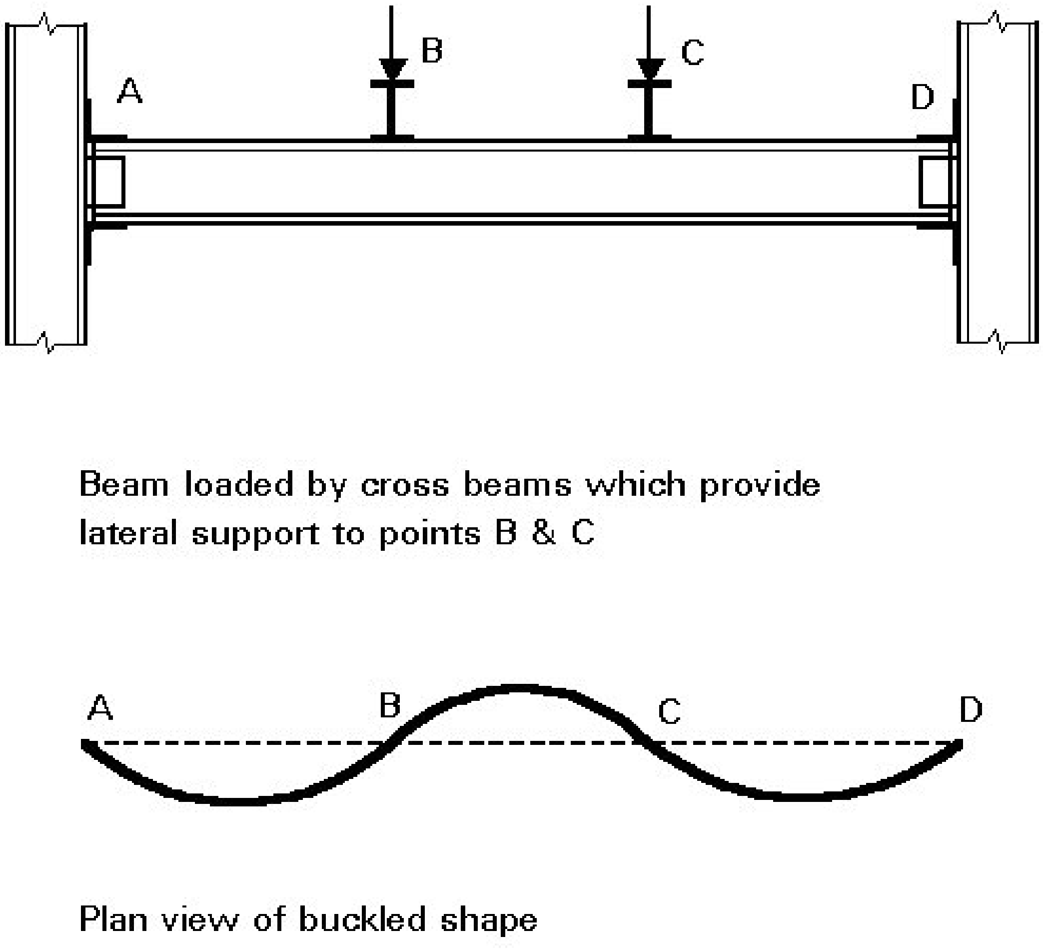


Figure 8 Buckling of beams provided with lateral bracing

6. DESIGN APPLICATION

Direct use of the theory of lateral-torsional instability for design is inappropriate because:

- The formulae are too complex for routine use, e.g. Equation (17) of [Lecture 7.9.2](#).
- Significant differences exist between the assumptions which form the basis of the theory and the characteristics of real beams. Since the theory assumes elastic behaviour, it provides an upper bound on the true strength (this point is discussed in general terms in [Lecture 6.6.2](#)).

Figure 9 compares a typical set of lateral-torsional buckling test data obtained using actual hot-rolled sections with the theoretical elastic critical moments given

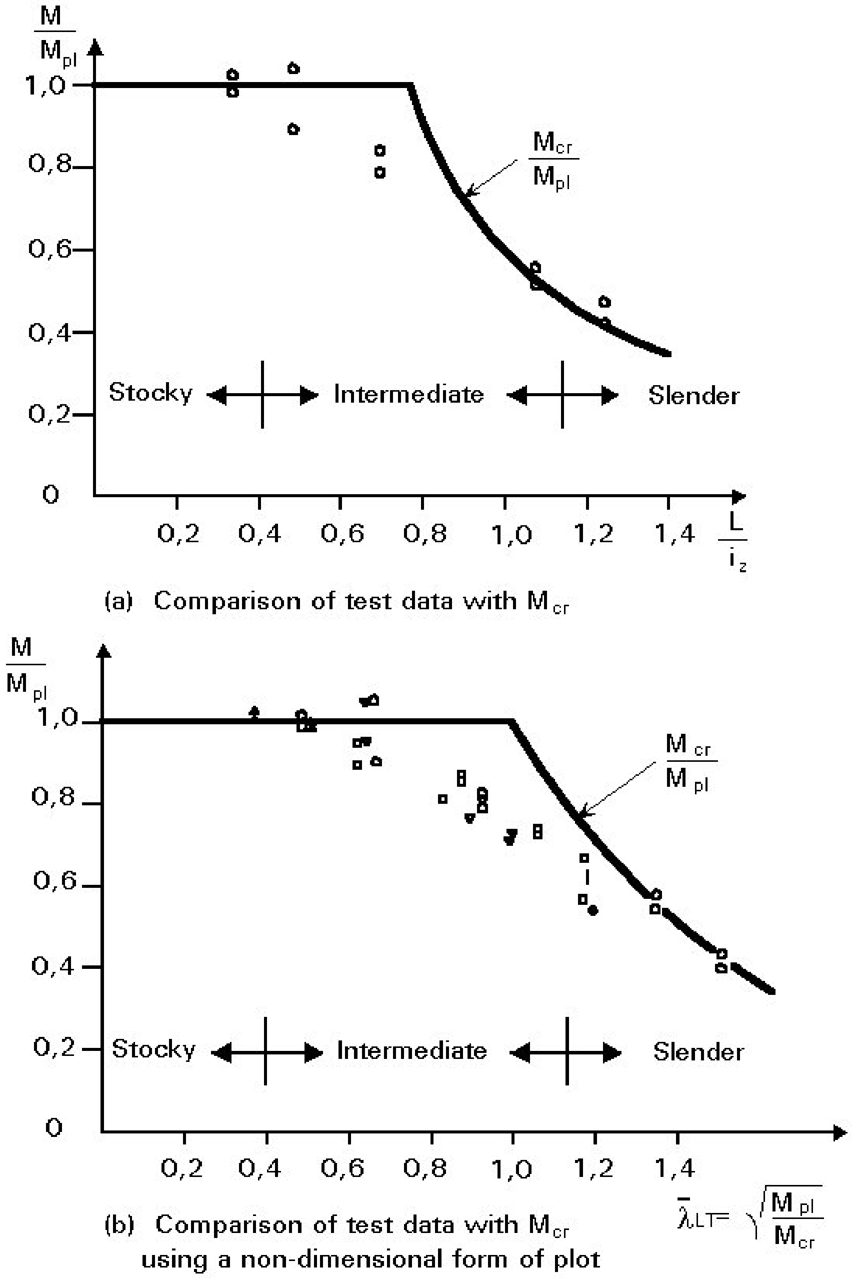


Figure 9 Comparison of test data with theoretical elastic critical moments

6. DESIGN APPLICATION

In Figure 9a only one set of data for a narrow flanged beam section is shown. The use of the λ_{LT} non-dimensional format in Figure 9b has the advantage of permitting results from different test series (using different cross-sections and different material strengths) to be compared directly. In both figures three distinct regions of behaviour can be observed:

- Stocky beams which are able to attain M_{pl}, with values of λ_{LT} below about 0,4 in Figure 9b.
- Slender beams which fail at moments close to M_{cr}, with values of λ_{LT} above 1,2 in Figure 9b.
- Beams of intermediate slenderness which fail to reach either M_{pl} or M_{cr}, with $0,4 < \lambda_{LT} < 1,2$ in Figure 9b.

Only in the case of beams in region 1 does lateral stability not influence design. For beams in region 2, which covers much of the practical range of beams without lateral restraint, design must be based on considerations of inelastic buckling suitably modified to allow for geometrical imperfections, residual stresses, etc.,. Thus both theory and tests must play a part, with the inherent complexity of the problem being such that the final design rules are likely to involve some degree of empiricism.

6. DESIGN APPLICATION

Section 7 outlines the provisions of Eurocode 3 [1] with regard to beam design, assuming typical sections as shown in Figure 10a and 10b. It should be noted that sections of the type illustrated in Figure 10b, with one axis of symmetry, e.g.channels, may only be included if the section is bent about the axis of symmetry, i.e. loads are applied through the shear centre parallel to the web of the channel. Singly-symmetrical sections bent in the other plane, e.g. an unequal flanged I-section bent about its major-axis as shown in Figure 10c, may only be treated by an extended version of the theory, principally because the section's shear centre no longer lies on the neutral axis.

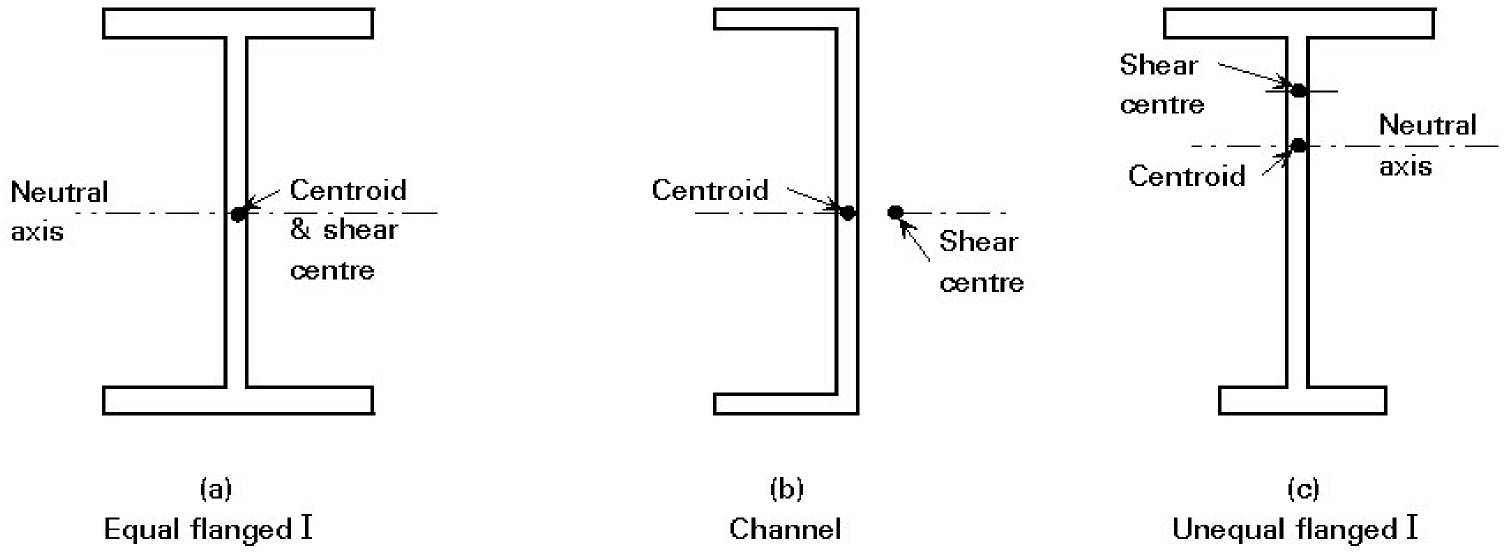


Figure 10 Equal flanged section and examples of sections with one axis of symmetry

7. METHOD OF EUROCODE 3

The buckling resistance moment [1] is given by:

$M_{bRd} = \chi_{LT} M_{Rd}$ (1)

where M_{Rd} is the moment resistance of the cross-section

χ_{LT} is the reduction factor for lateral-torsional buckling

In determining M_{Rd} the section classification should, of course, be noted and the appropriate section modulus used in conjunction with the material design strength f_d. The value of χ_{LT} depends on the beam's slenderness λ_{LT} and is given by:

$\chi_{LT} = 1/ \{\phi_{LT} + [\phi_{LT}^2 - \lambda_{LT}^2]^{1/2}\}$ (2)

where $\phi_{LT} = 0,5 [1 + \alpha_{LT}(\lambda_{LT} - 0,20) + \lambda_{LT}^2]$

and $\alpha_{LT} = 0,21$ for rolled sections

$\alpha_{LT} = 0,49$ for welded beams

7. METHOD OF EUROCODE 3

Figure 11 illustrates the relationship between c_{LT} and λ_{LT}, showing how it follows the pattern of behaviour exhibited by the test data of Figure 9.
When λ_{LT} <= 0,4, the value of χ_{LT} is sufficiently close to unity that design may be based on the full resistance moment M_{Rd}.

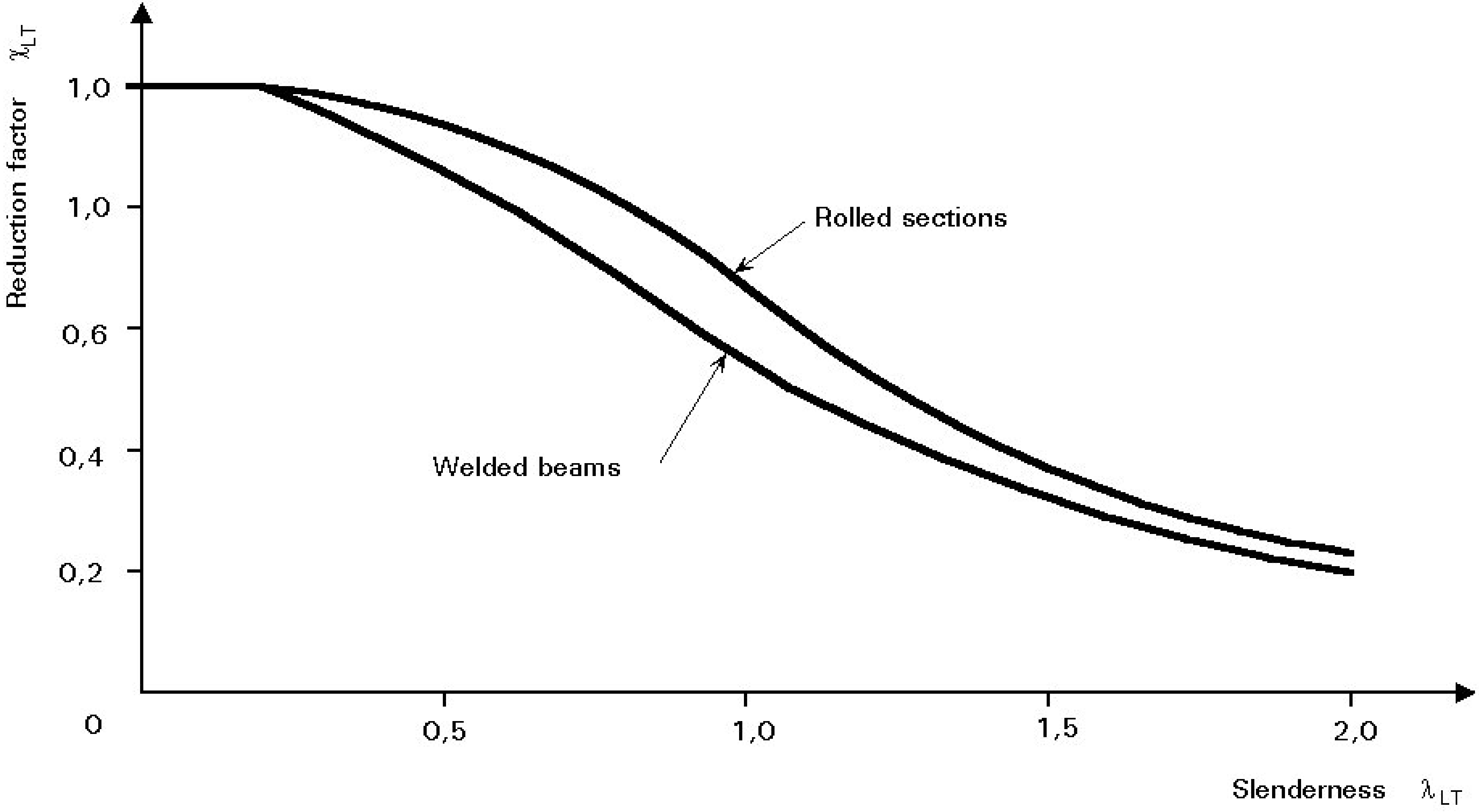


Figure 11 Lateral-torsional buckling reduction factor

7. METHOD OF EUROCODE 3

The slenderness λ_{LT}, which is a measure of the extent to which lateral-torsional buckling reduces a beam's load carrying resistance, is a function of M_{Rd} and M_{cr}. M_{cr} is the elastic critical buckling moment, a quantity similar in concept to the Euler load for a strut since it is derived from a theory (see Lecture 7.9.2) that assumes "perfect" behaviour, i.e. an initially straight member, elastic response, no misalignment of the loading, etc..

Thus λ_{LT} is taken as:

$\lambda_{LT} = [M_{Rd} / M_{cr}]1/2$ (3)

For calculation purposes Equation (3) may be rewritten as:

$\lambda_{LT} = [\lambda_{LT} / \lambda_1]$ (4)

where $\lambda_1 = \pi[E/f_y]^{1/2}$

$= 93{,}9[235/f_y]^{1/2}$

where

$$\lambda_{LT} = \frac{L/i_z}{[C_1]^{1/2}\left[1+\frac{1}{20}\left(\frac{L/i_z}{d/t_f}\right)^2\right]^{1/4}} \qquad (5)$$

and ψ is the end moment ratio defined in Figure 13.

Figure 13 Moment gradient loading over beam span L

7. METHOD OF EUROCODE 3

Taking as an example the end span of a continuous beam for which y = 0 gives C_1=1,75 and thus λ_{LT} will be reduced to 0,76 (= 1/ √1,75) of the value for uniform moment, leading to an increase in χ_{LT} and thus in M_{bRd}.

Variations in the conditions of lateral restraint may be treated by introducing k-coefficients to modify the geometrical length L into kL when determining M_{cr}. For conditions with more restraint, values of k < 1,0 are appropriate, leading to an increase in M_{cr} and thus, via a reduction in λ_{LT} , to increases in χ_{LT} and M_{bRd}.

7. METHOD OF EUROCODE 3

Similarly additional C-coefficients may be used directly in the determination of M_{cr} to provide modified values of λ_{LT} appropriate for a wide range of load types. In particular, this method should be used to calculate the reduced M_{cr} appropriate for destabilising loads. These are loads that act above the level of the beam's shear centre and are free to move sideways with the beam as it buckles, as shown in Figure 14.

For cross-sections of the type illustrated in Figure 9c, for which the shear centre and centroid do not lie on the same horizontal axis, evaluation of M_{cr} becomes more complex.

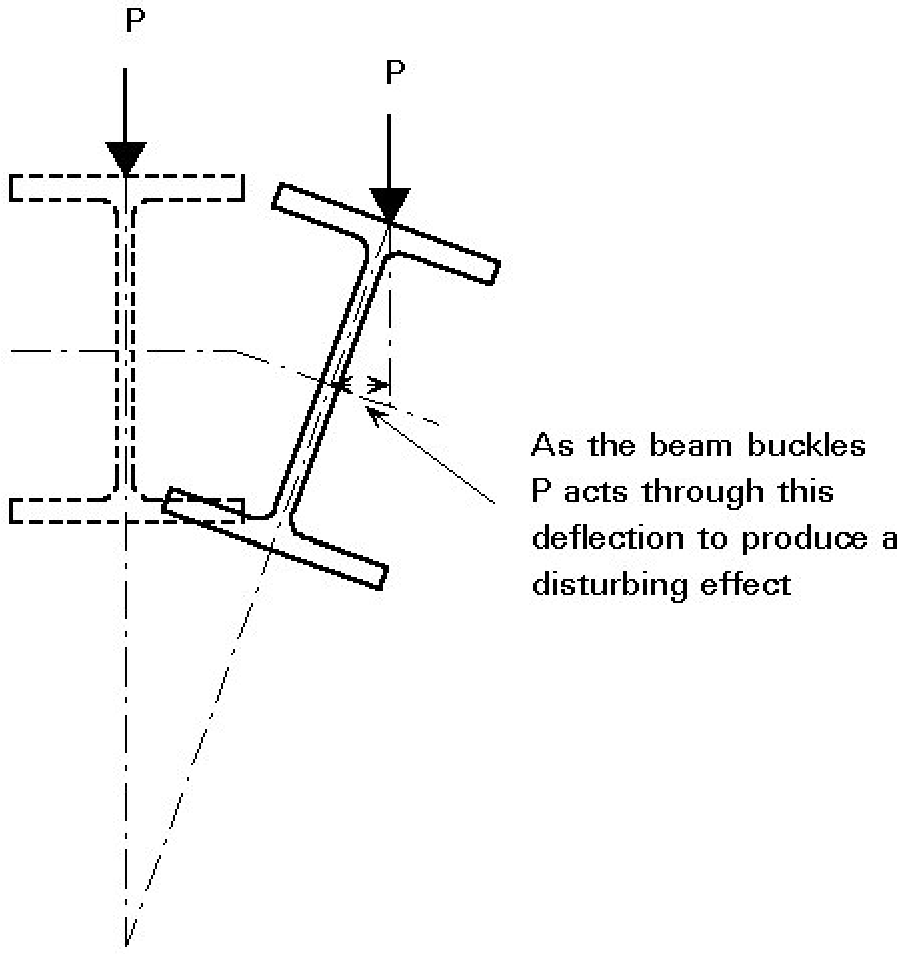


Figure 14 Destabilising loading

7. METHOD OF EUROCODE 3

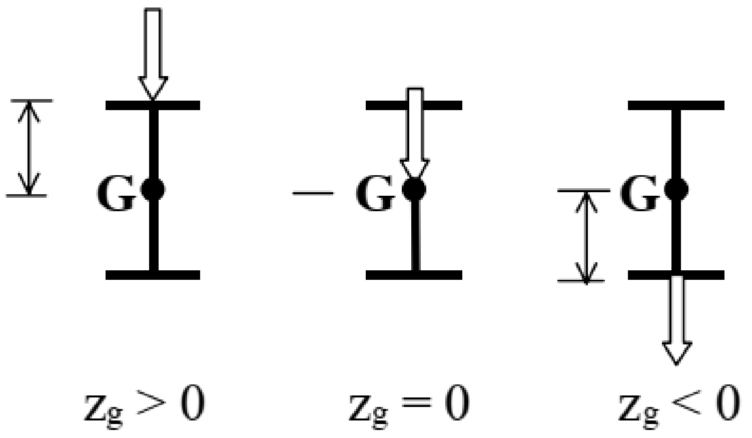
How to obtain a M_{cr}?

- by using formula:

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{L^2}\left[\sqrt{\frac{I_w}{I_z} + \frac{L^2 G I_t}{\pi^2 E I_z} + (C_2 z_g)^2} - C_2 z_g\right]$$

where

zg – distance beetwen point aplication and the centre of the gravity of the beam



7. METHOD OF EUROCODE 3

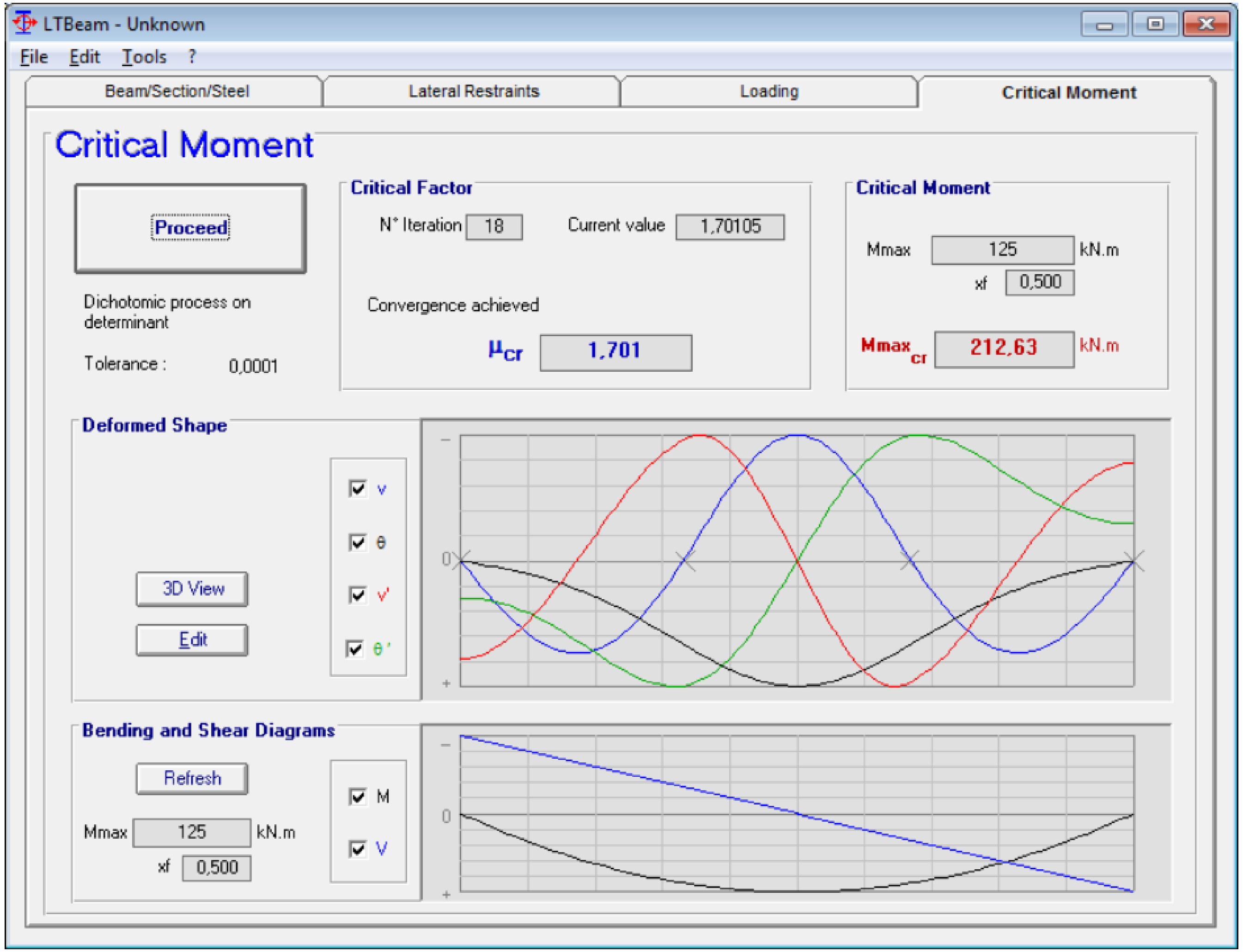
C1, C2 – coefficients responsible for adjusting formula to real bending moment distribution.

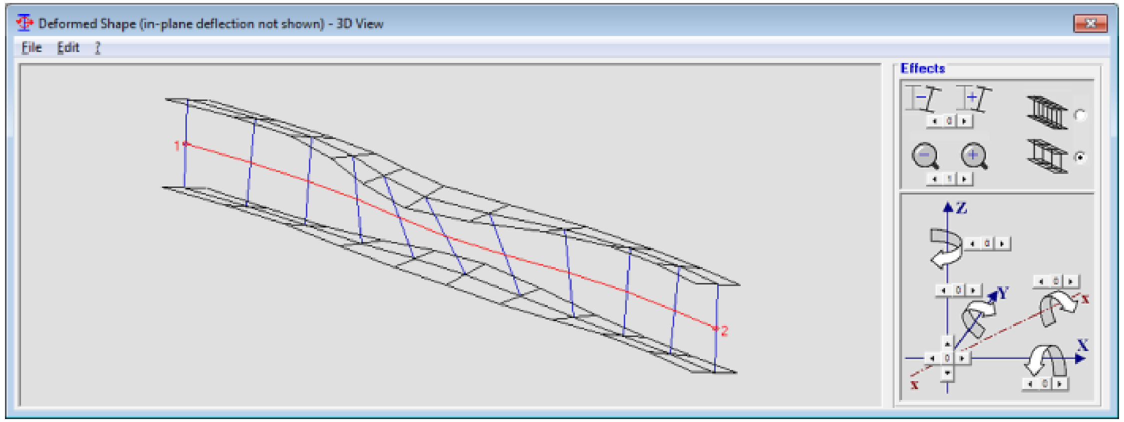
C_1	ψ	-1	-0,9	-0,8	-0,7	-0,6	-0,5	-0,4	-0,3	-0,2	-0,1	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
$\mu = qL^2/8M > 0$	0	2,554	2,627	2,606	2,534	2,438	2,331	2,219	2,104	1,990	1,878	1,770	1,667	1,569	1,477	1,391	1,312	1,238	1,171	1,109	1,052	1,000
	0,1	2,450	2,411	2,337	2,246	2,148	2,046	1,943	1,842	1,744	1,648	1,558	1,472	1,391	1,315	1,245	1,179	1,119	1,070	1,037	1,018	1,012
	0,2	2,233	2,160	2,076	1,986	1,894	1,802	1,712	1,625	1,541	1,461	1,385	1,314	1,246	1,187	1,139	1,101	1,071	1,049	1,034	1,025	1,022
	0,3	2,003	1,925	1,843	1,760	1,678	1,598	1,521	1,446	1,375	1,310	1,254	1,206	1,165	1,131	1,102	1,080	1,062	1,048	1,039	1,033	1,030
	0,4	1,790	1,717	1,642	1,569	1,497	1,430	1,370	1,316	1,269	1,227	1,190	1,159	1,131	1,108	1,089	1,073	1,061	1,051	1,044	1,039	1,037
	0,5	1,604	1,539	1,479	1,423	1,373	1,326	1,284	1,247	1,213	1,184	1,157	1,135	1,115	1,098	1,084	1,072	1,063	1,055	1,049	1,046	1,043
	0,6	1,468	1,421	1,377	1,336	1,299	1,265	1,234	1,206	1,181	1,159	1,139	1,122	1,106	1,093	1,082	1,073	1,065	1,059	1,054	1,051	1,049
	0,7	1,382	1,346	1,313	1,282	1,253	1,227	1,203	1,181	1,161	1,144	1,128	1,114	1,102	1,091	1,082	1,074	1,068	1,063	1,059	1,056	1,054
	0,8	1,324	1,296	1,270	1,245	1,222	1,201	1,182	1,164	1,148	1,134	1,121	1,110	1,100	1,090	1,083	1,076	1,071	1,066	1,062	1,060	1,058
	0,9	1,284	1,261	1,239	1,219	1,201	1,183	1,167	1,153	1,140	1,128	1,117	1,107	1,098	1,091	1,084	1,078	1,073	1,069	1,066	1,063	1,061
	1	1,254	1,236	1,217	1,201	1,185	1,170	1,157	1,145	1,133	1,123	1,114	1,105	1,098	1,091	1,085	1,080	1,076	1,072	1,069	1,067	1,065
	1,1	1,233	1,217	1,201	1,187	1,174	1,161	1,150	1,139	1,129	1,120	1,112	1,105	1,098	1,092	1,087	1,082	1,078	1,075	1,072	1,070	1,068
	1,2	1,216	1,202	1,189	1,176	1,165	1,154	1,144	1,135	1,126	1,118	1,111	1,104	1,098	1,093	1,088	1,084	1,081	1,077	1,075	1,072	1,071
	1,3	1,203	1,191	1,179	1,168	1,158	1,148	1,139	1,131	1,124	1,117	1,110	1,104	1,099	1,094	1,090	1,086	1,083	1,079	1,077	1,075	1,073
	1,4	1,193	1,181	1,172	1,162	1,153	1,144	1,136	1,129	1,122	1,116	1,110	1,104	1,099	1,095	1,091	1,087	1,084	1,081	1,079	1,077	1,075
	1,5	1,184	1,175	1,165	1,157	1,148	1,141	1,134	1,127	1,121	1,115	1,110	1,105	1,100	1,096	1,092	1,089	1,086	1,083	1,081	1,079	1,078
	1,6	1,177	1,168	1,160	1,152	1,145	1,138	1,131	1,125	1,120	1,114	1,110	1,105	1,101	1,097	1,094	1,091	1,088	1,085	1,083	1,081	1,080
	1,7	1,171	1,164	1,156	1,149	1,142	1,135	1,130	1,124	1,119	1,114	1,109	1,105	1,102	1,098	1,095	1,092	1,089	1,087	1,085	1,083	1,081
	1,8	1,167	1,159	1,153	1,146	1,140	1,134	1,128	1,123	1,118	1,114	1,109	1,106	1,102	1,099	1,096	1,093	1,090	1,088	1,086	1,084	1,083
	2	1,159	1,153	1,147	1,141	1,136	1,131	1,126	1,122	1,118	1,114	1,110	1,107	1,103	1,101	1,098	1,095	1,093	1,091	1,089	1,087	1,086
	2,2	1,153	1,148	1,143	1,138	1,133	1,129	1,125	1,121	1,117	1,114	1,111	1,107	1,105	1,102	1,100	1,097	1,095	1,093	1,091	1,090	1,089
	2,5	1,148	1,143	1,139	1,135	1,131	1,127	1,124	1,120	1,117	1,114	1,111	1,109	1,107	1,104	1,102	1,100	1,098	1,096	1,094	1,093	1,092
	3	1,141	1,138	1,135	1,131	1,128	1,126	1,123	1,120	1,117	1,115	1,113	1,111	1,109	1,107	1,105	1,103	1,102	1,100	1,099	1,098	1,096
	3,5	1,137	1,134	1,132	1,130	1,127	1,125	1,122	1,120	1,118	1,116	1,114	1,112	1,111	1,109	1,108	1,106	1,105	1,103	1,102	1,101	1,100
	4	1,135	1,133	1,130	1,128	1,126	1,124	1,122	1,121	1,119	1,117	1,115	1,114	1,112	1,111	1,110	1,108	1,107	1,105	1,105	1,104	1,103
	5	1,132	1,130	1,129	1,127	1,126	1,124	1,122	1,121	1,119	1,118	1,117	1,116	1,115	1,114	1,112	1,111	1,110	1,109	1,108	1,108	1,107
	7	1,129	1,128	1,127	1,126	1,125	1,124	1,123	1,122	1,121	1,120	1,120	1,119	1,118	1,117	1,116	1,115	1,114	1,114	1,113	1,112	1,112
	10	1,128	1,127	1,127	1,126	1,125	1,125	1,124	1,123	1,123	1,122	1,121	1,121	1,120	1,119	1,119	1,118	1,118	1,117	1,117	1,116	1,116
	∞	1,127	1,127	1,127	1,127	1,127	1,127	1,127	1,127	1,127	1,127	1,127	1,127	1,127	1,127	1,127	1,127	1,127	1,127	1,127	1,127	1,127

TABLEAU 1 – *Coefficient C_1 – Moments d'extrémité et charge répartie – $\mu > 0$*

7. METHOD OF EUROCODE 3

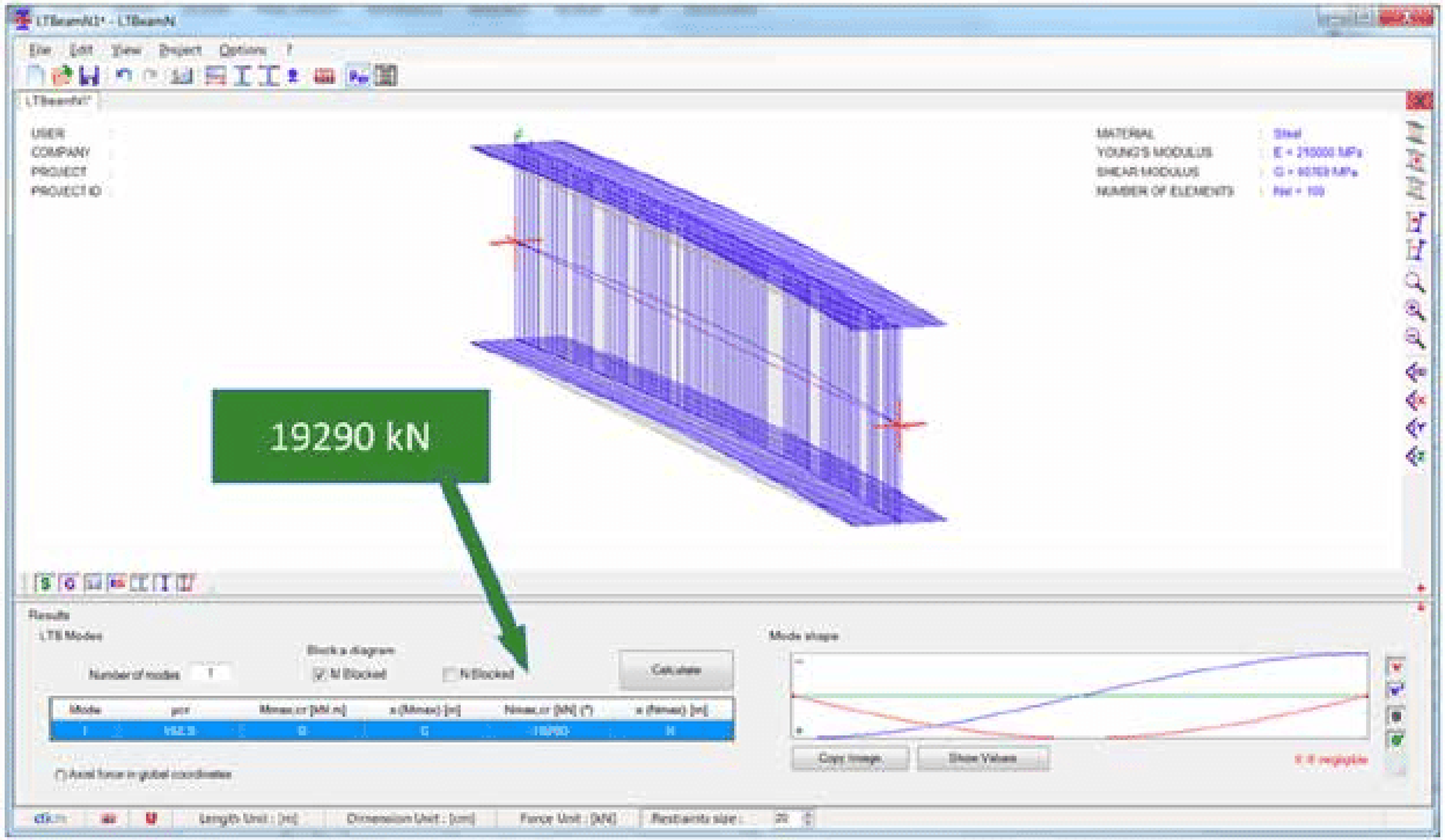
- by using numerical analysis:
  - LTBeam





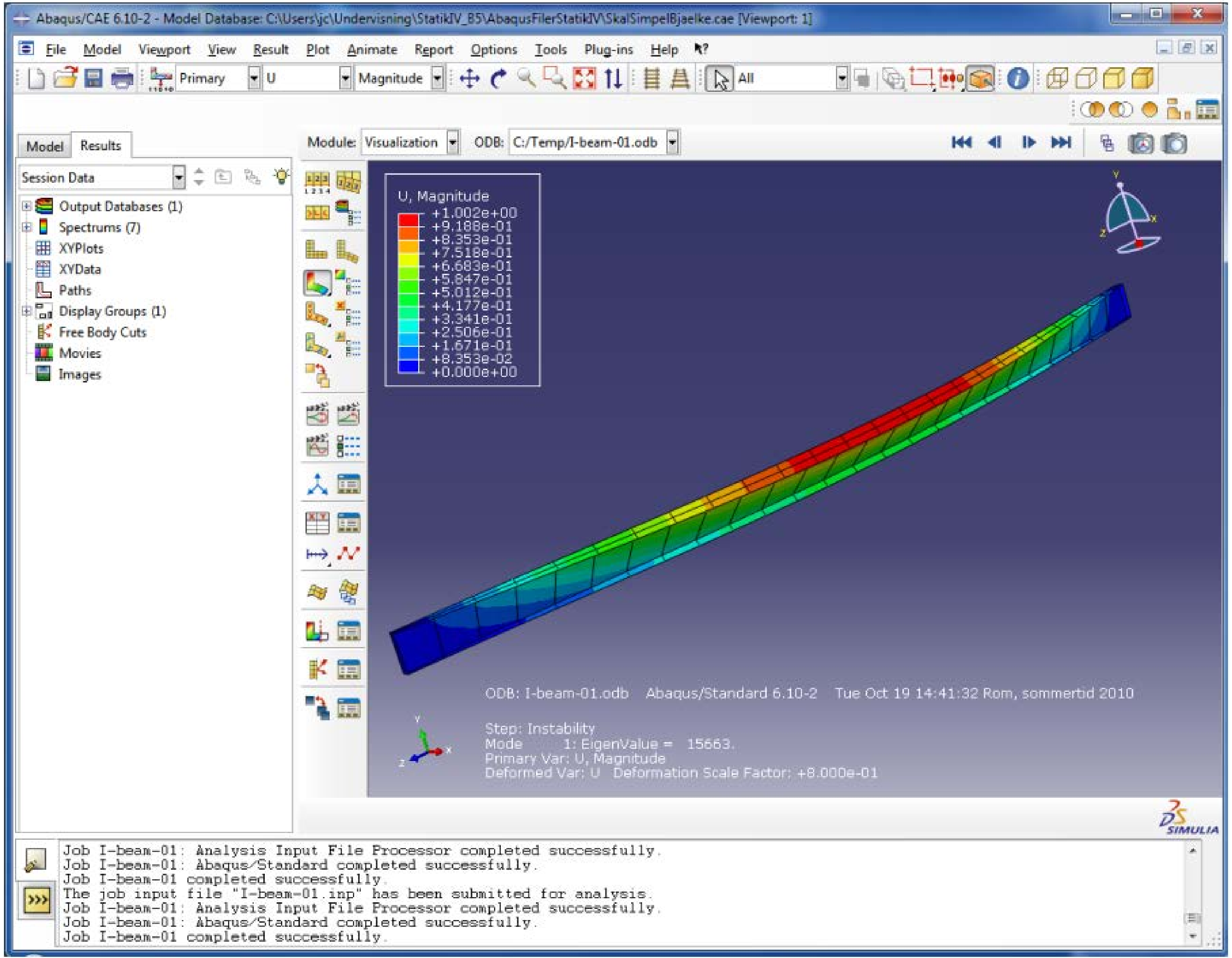
7. METHOD OF EUROCODE 3

- by using numerical analysis:
  - LTBeamN



7. METHOD OF EUROCODE 3

- by using numerical analysis:
  - advanced numerical systems (Abaus, Ansys,...)



8. CONCLUDING SUMMARY

- Beams that are not restrained along their length and are bent about their strong axis are subject to lateral torsional buckling.
- Unbraced span, lateral slenderness (L/i_z), cross-sectional shape (torsional and warping rigidities), moment distribution and end restraint are the primary influences on buckling resistance.
- Bracing of sufficient stiffness and strength, that restrains either torsional or lateral deformations, may be used to increase buckling resistance.
- Although elastic critical load theory provides a background for understanding the behaviour of laterally unrestrained beams, it requires both simplifications and empirical modification if it is to form a suitable basis for a design approach.
- In order to check the lateral buckling resistance of a trial section, its effective slenderness λ_{LT} must first be obtained.
- Variation in either lateral support conditions or the form of the applied loading may be accommodated in the design process by means of coefficients k and C, used to modify either the basic slenderness λ_{LT} or the basic elastic critical moment M_{cr}.

9. REFERENCES

[1] Eurocode 3: "Design of Steel Structures": ENV 1993-1-1: Part 1.1: General rules and rules for buildings, CEN, 1992.

10. ADDITIONAL READING

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   Chapter 3 deals with laterally unrestrained beams.
3. Timoshenko, S. P. and Gere, J. M., "Theory of Elastic Stability" Second Edition, McGraw Hill 1961.
   Basic derivations for the elastic critical moment for a variety of beam problems are provided in Chapter 6.
4. Bleich, F., "Buckling Strength of Metal Structures", McGraw Hill 1952.
   Chapter 4 presents the basic theory of lateral buckling of beams.
5. Galambos, T. V., "Structural Members and Frames", Prentiss Hall 1968.
   Chapter 2 deals with the fundamentals of elastic behaviour, whilst Chapter 3 covers elastic and inelastic behaviour and design of laterally unrestrained beams.
6. Trahair, N. S. and Bradford, M. A., "The Behaviour and Design of Steel Structures", Chapman and Hall, Second Edition, 1988.
7. Laterally unrestrained beams are dealt with in Chapter 6.